Exam III

1. Show that $(X^\circ)^\circ = X^\circ$ is open, where X° is the set of all interior points of X. Conclude that X° is an open set.

2. Given a point $a \in X \subseteq \mathbb{R}$. We call a a boundary point of X if for every open interval I containing a, we have $I \cap X \neq \emptyset$ and $I \cap (\mathbb{R} - X) \neq \emptyset$. The set of all boundary points, ∂X, is called the *boundary* of X. Show that a set A is open if and ony if $A \cap \partial A = \emptyset$.

3. Let $X \subseteq \mathbb{R}$. Show that $\overline{X} = X \cup \partial X$. Conclude that X is closed if and only if $\partial X \subseteq X$.

4. Show that finite union of compact sets is compact, and also arbitrary intersection of compact sets is compact as well.

5. Show that every compact set X, satisfying $X' = \emptyset$, is finite. Where X' is the derived set of X, i.e. the collection of all accumulation points.

Extra (2 pts). Show that the sum of the lengths of removed intervals in the construction of the Cantor set is one. Where by length of (a, b), I mean $b - a$. *[Hint: The formula for the sum of terms of a geometric progression $a, ar, ar^2, \ldots$ with $|r| < 1$ is $\frac{a}{1-r}$]*