Exam I

1. Prove that there is no largest negative rational number.

   Solution. If $N \in \mathbb{Q}$ is the largest negative rational number then $N < \frac{N}{2}$, a contradiction. $\square$

2. Prove that if x and y are positive real numbers, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.

   Solution. Suppose $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ and take $x = y = 1$ to obtain $\sqrt{2} = 2$, a contradiction. $\square$

3. Prove that n is even if and only if $n^2 + 1$ is odd.

   Solution. Suppose $n = 2k$, then $n^2 + 1 = 4k^2 + 1$, an odd number. Conversely, if $n = 2k + 1$, then $n^2 + 1 = 4k^2 + 4k + 2 = 2(2k^2 + 2k + 1)$, an even number. $\square$

4. Give 2 examples of implications, and for each write down their contrapositive. Have one to be real-world example, and the other to be a math example.

   Solution. The any prime that is even is divisible by 2, whose contrapositive is: Any prime number that is not divisible by two is odd.

   If I don't think nothing is going on then I shouldn't talk. Whose contrapositive is: If I should talk then I think something is going on. $\square$

5. Are there infinitely many composite numbers? Prove your answer.

   Solution. Suppose that are finitely many composite numbers, let M be the greatest of them. Then $2M$ is composite and greater than M, a contradiction. $\square$