Monday, September 23, 2024 12:39 PM

$X = \{x ; x \text{ satisfies some property}\}$

OR

$\{x \mid \quad " \quad \}$

$\mathbb{N} = \{1, 2, 3, ..\}$

3.5 (a) $\{3, 5, 7, 9, 11, ...\}$

$\{2n+1 ; n \in \mathbb{N}\}$

$\{2n+1 \mid n \in \mathbb{N}\}$

$\{x ; x \text{ is odd and } x \geq 3\}$

(c) $\{-2, -1, 0, 1, 2, 3, 4, 5\}$

$\{x \in \mathbb{Z} ; -2 \leq x \leq 5\}$

(b) $\{..., -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, ...\}$

$\{\frac{n\pi}{2} ; n \in \mathbb{Z}\}$

= "The set of all numbers of the form "$\frac{n\pi}{2}$" such that "n" is an integer"

$|X|$ ~~$card(\mathbb{N}) = \aleph_0$~~

Cardinality $\Leftrightarrow$ "number of elements"

3.6 (a) $|\{a, b, d\}| = 3$

(b) $|\{\{1\}, 3, \{\{1\}, 3\}\}| = 3$

(c) $|\{\{1, 2, 3\}\}| = 1$

(d) $X: \{s, e, t\} \times \{t, h, e, o, r, y\}$

$A \times B = \{(a, b) ; a \in A \text{ and } b \in B\}$

$X = \{(s,t), (s,h), (s,e), ...$

$(e,t), ..$

$(t,t), (t,h), (t,e) .... \}$

$= \{(a,b) ; a \in \{s,e,t\} \text{ and } b \in \{t,h,e,o,r,y\}\}$

(e) $|P(\{1,2,3\})| = 8$ "Power set of X, denoted by P(X) is the set of all subsets"

$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

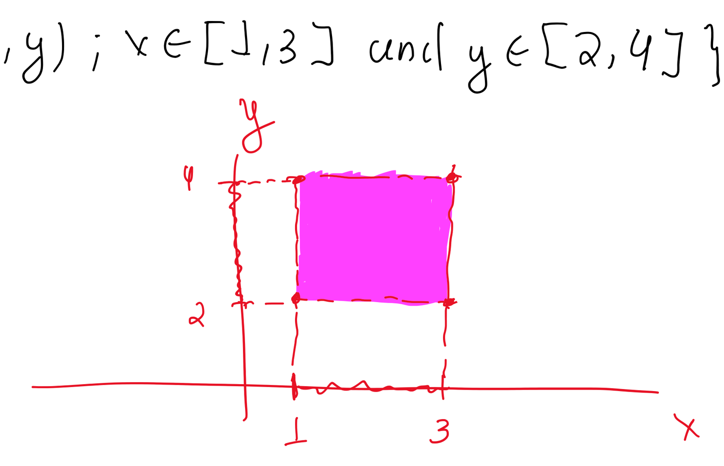
$|P(\{1, ..., n\})| = 2^n$

(f) $P(P(\{a,b\})) = P(\{\emptyset, \{a\}, \{b\}, \{a,b\}\})$

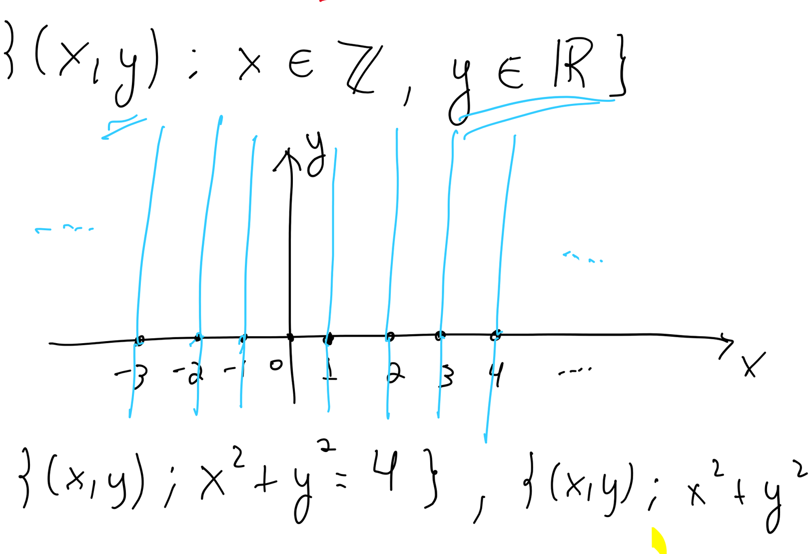
$= \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{\{a\}, \{a,b\}\}$

$\{\{b\}, \{a,b\}\}, \{\{a\}, \{b\}\}, \{\{a\}, \{b\}, \{a,b\}\}$

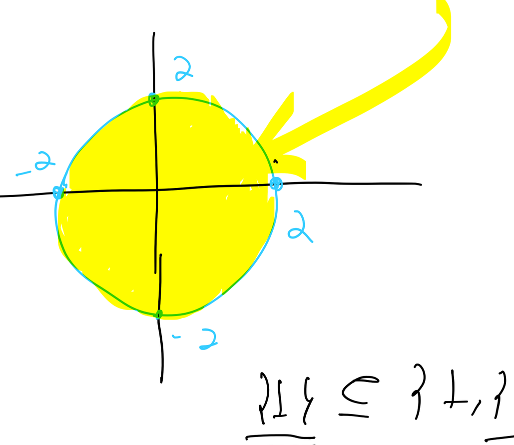
3.7 (a) $\{(x,y) ; x \in [1,3] \text{ and } y \in [2,4]\}$



(b) $\{(x,y) : x \in \mathbb{Z}, y \in \mathbb{R}\}$



(c) $\{(x,y) ; x^2 + y^2 = 4\}$, $\{(x,y) ; x^2 + y^2 \leq 4\}$



3.8 T or F $\quad \{1\} \subseteq \{1, \{1\}\}$

(a) $1 \in \{1, \{1\}\}$ T

(b) $1 \subseteq \{1, 2\}\}$ F

(c) $1 \in P(\{1, \{1\}\})$ F

"Elements of the power sets are sets themselves ($\{1\} \in P(\{1, \{1\}\})$ ✓)

(d) $\{1\} \in \{1, \{1\}\}$ T $\quad X \subseteq Y$

(e) $\{1\} \subseteq \{1, \{1\}\}$ T $\quad x \in X \Rightarrow x \in Y$

(f) $\{1\} \in P(\{1, \{1\}\})$, T

(g) $\emptyset \in \mathbb{N}$ F

(k) $\emptyset \subseteq \mathbb{N}$ T $\quad \mathbb{Q} \subseteq \mathbb{R}$

(l) $\emptyset \in P(\mathbb{N})$ T

(m) $\mathbb{Q} \times \mathbb{Q} \subseteq \mathbb{R} \times \mathbb{R}$ T

(n) $\mathbb{R}^2 \subseteq \mathbb{R}^3$ F

$(a,b) \quad (a,b,c)$

(o) $\emptyset \subseteq \{1,2,3\} \times \{a,b\}$ T

3.9. $P(\emptyset) = \emptyset$

(b) $P(\{\mathbb{N}, \mathbb{Q}, \mathbb{R}\}) = \{\emptyset, \mathbb{N}, \mathbb{Q}, \mathbb{R}, \{\mathbb{N}, \mathbb{Q}\}, \{\mathbb{N}, \mathbb{R}\}, \{\mathbb{Q}, \mathbb{R}\}, \{\mathbb{N}, \mathbb{Q}, \mathbb{R}\}\}$

3.10. $\{5a + 3b ; a, b \in \mathbb{Z}\} = \mathbb{Z}$

$a = 0 \quad \{3b ; b \in \mathbb{Z}\} \quad \{0, \pm 3, \pm 6, \pm 9, ..\}$

$b = 0 \quad \{5a ; a \in \mathbb{Z}\} \quad \{0, \pm 5, \pm 10, ..\}$

$a = b \quad \{8a ; a \in \mathbb{Z}\} \quad \{0, \pm 8, \pm 16, ..\}$

$b = -a \quad \{2a ; a \in \mathbb{Z}\} \quad \{0, \pm 2, \pm 4, \pm 6, ..\}$

$\{2n+1 ; n \in \mathbb{Z}\} \quad \{\pm 1, \pm 3, ...\}$

$5a + 3b = 2n + 1$

$\begin{cases} a = n + p \\ b = -n + q \end{cases}$

$5n + 5p - 3n + 3q$

$2n + 5p + 3q$

$p = 2 \quad q = -3$

$2n + 10 - 9$

$2n + 1$

3.11 A, B, C sets

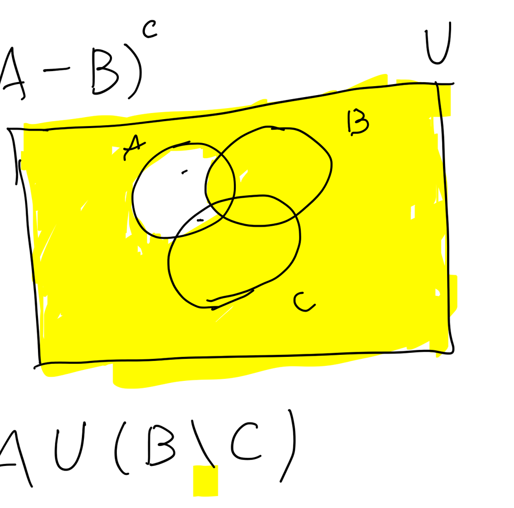
$(A \times B) \times C$ and $A \times (B \times C)$

$\{(x,y) ; x \in A \times B \text{ and } y \in C\}$ $\quad \{(x,y) ; x \in A \text{ and } y \in B \times C\}$

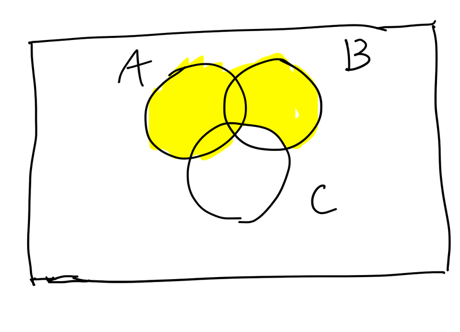
yes, there is a difference. The first coordinate of $(A \times B) \times C$ is in $(A \times B)$, whereas the first coordinate of $A \times (B \times C)$ is in A.

3.13 $A, B, C \subseteq U$

(a) $(A - B)^c$



(b) $A \cup (B \setminus C)$



(d) $A^c \cap (B \setminus C)$