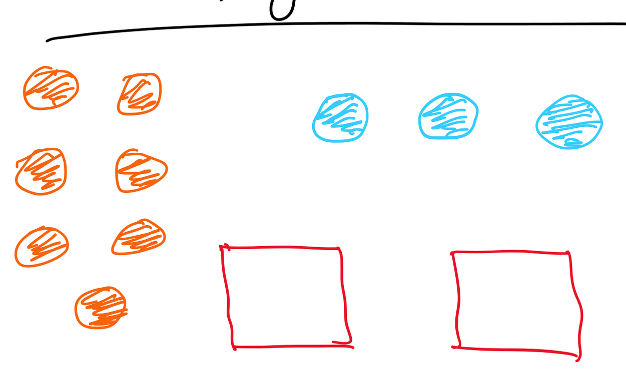
Wednesday, August 28, 2024 12:31 PM

The pigeonhole Principle



F ⟺ F
T ⟺ T

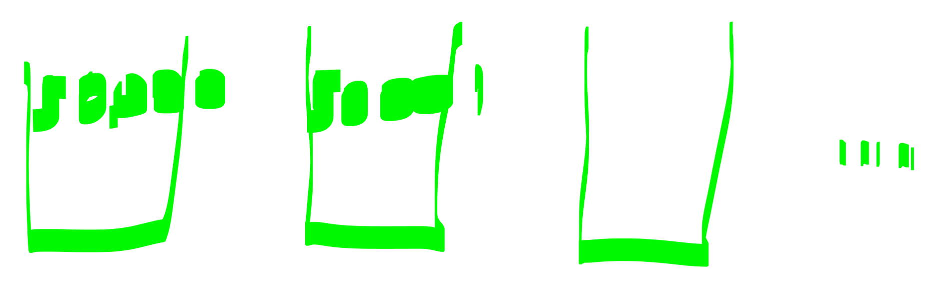
If one has "n+1" balls and "n" boxes, then if all the balls are to be put inside the "n" boxes then at least one box has 2 balls.

⇕ ⟺ : if and only if

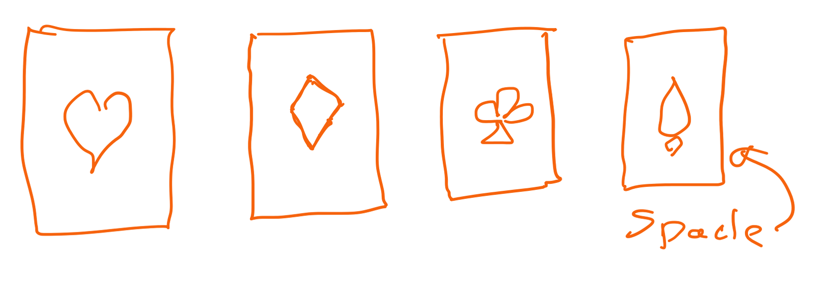
"Kn+1" balls, n boxes ⟹ at least one box "K+1" balls

Proposition: There are 3 non-balding people in Sacramento, CA, who have exactly the same number of hairs (on their head)

- A non-balding human has between 50000–199199 hairs.
- Sacramento has 380000 non-bal. people.

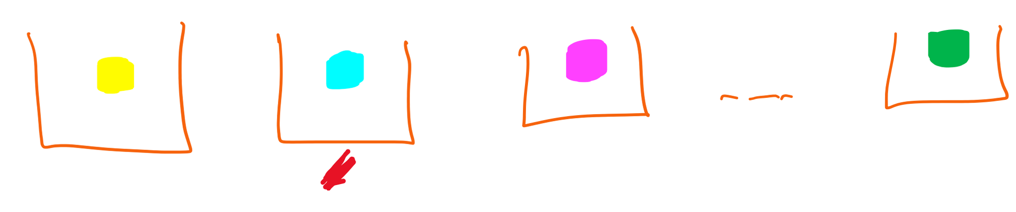


Ex: Among any 5 playing cards, there are at least 2 cards of the same suit.

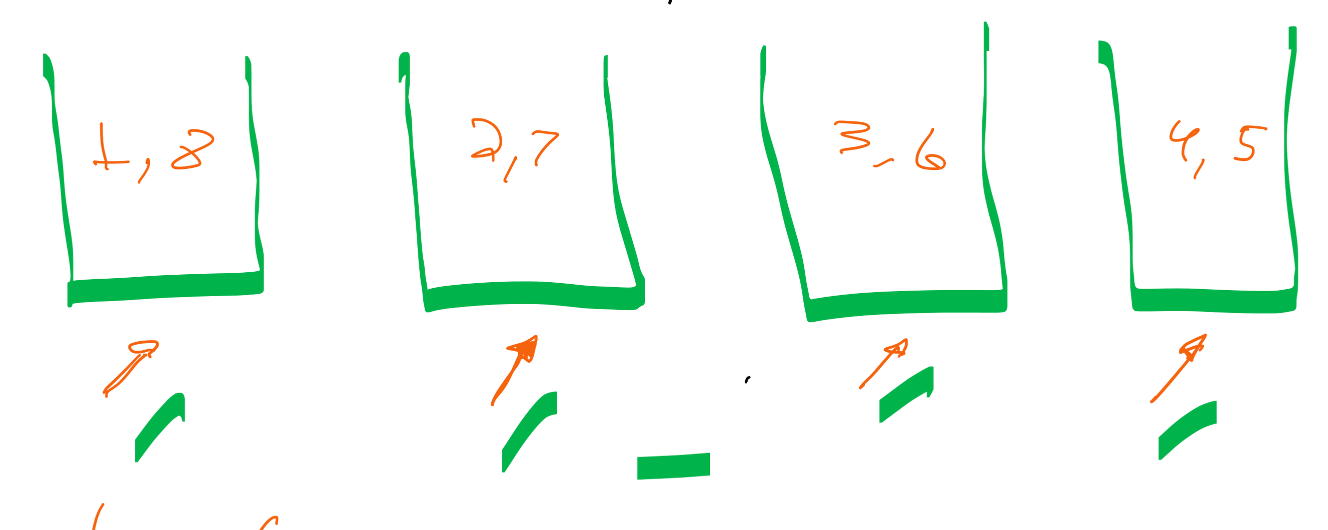


Ex: In a group of 13 people at least 2 of them will have birthday in the same month.

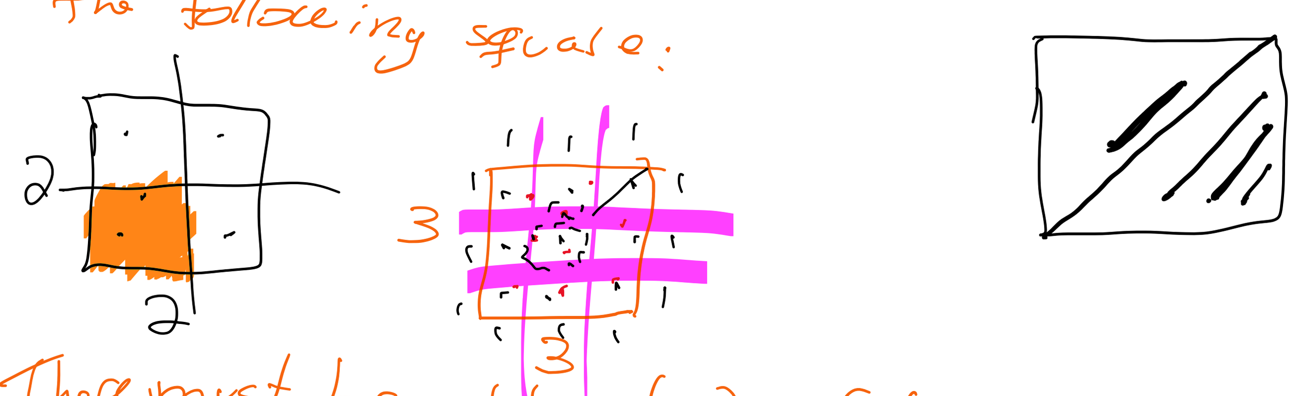
Ex: Socks



Proposition: Given any 5 numbers from the set {1, 2, 3, 4, 5, 6, 7, 8}, two of the chosen numbers will add up to 9.



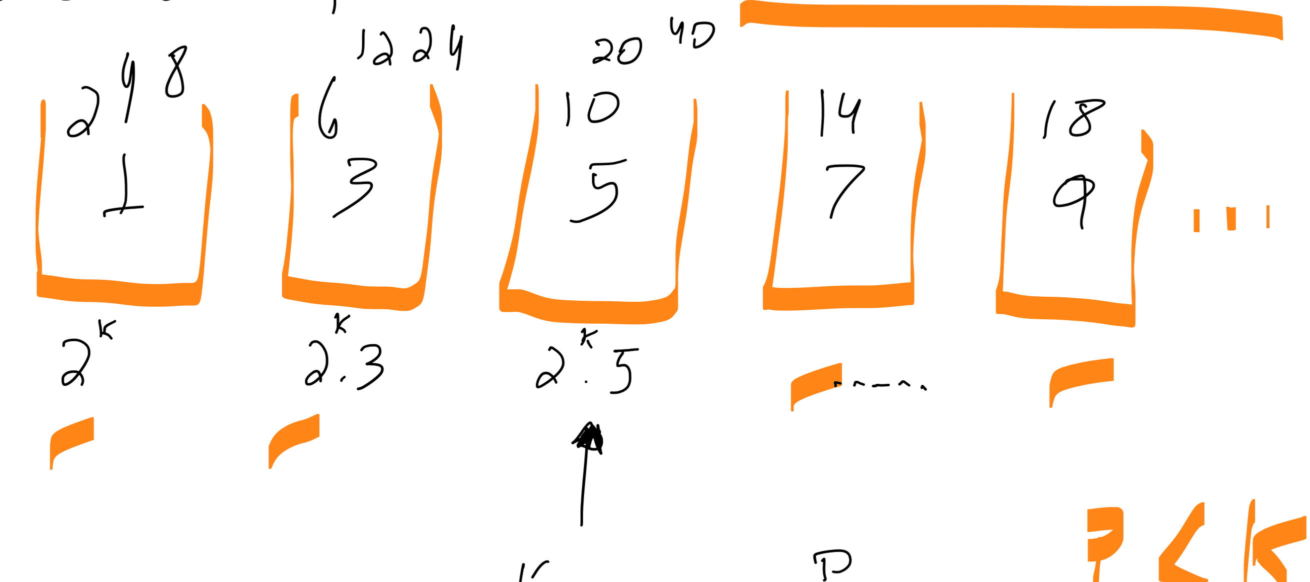
Proposition: Given any 10 points inside the following square:



There must be at least 2 of these points which are of distance at most $\sqrt{2}$ from each other.

Erdos' problem: $120 = 2 \cdot 60 = 2^2 \cdot 30 = 2^3 \cdot 15$

"Given any 101 integers from {1, 2, 3, ..., 200}, at least one of those numbers will divide another."

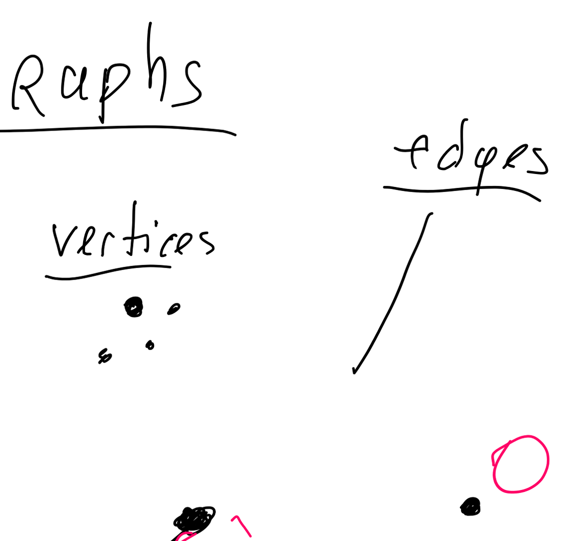


$2^k \cdot 5,\ 2^p \cdot 5$

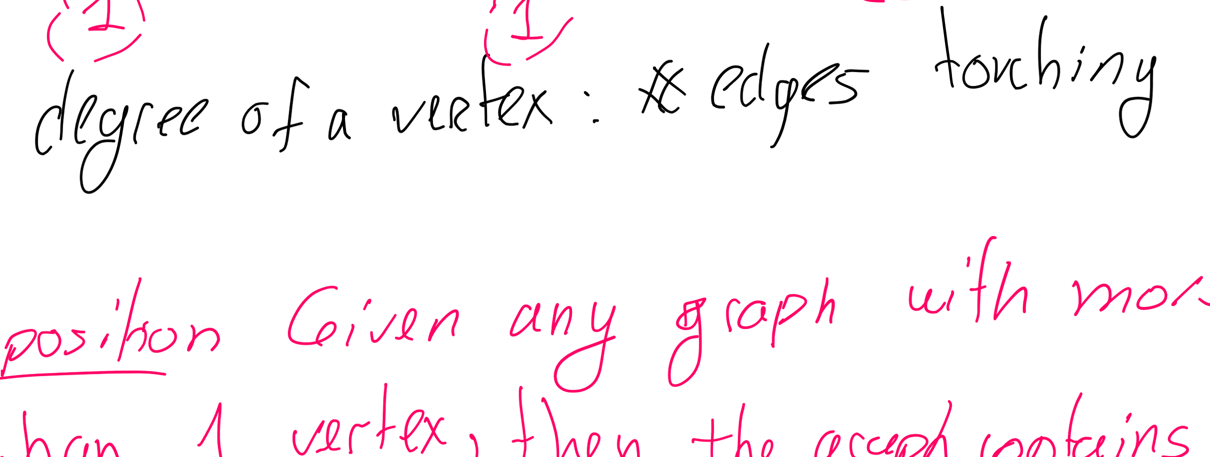
$p < k$

$\frac{2^k \cdot 5}{2^p \cdot 5} = 2^{k-p}$

Graphs



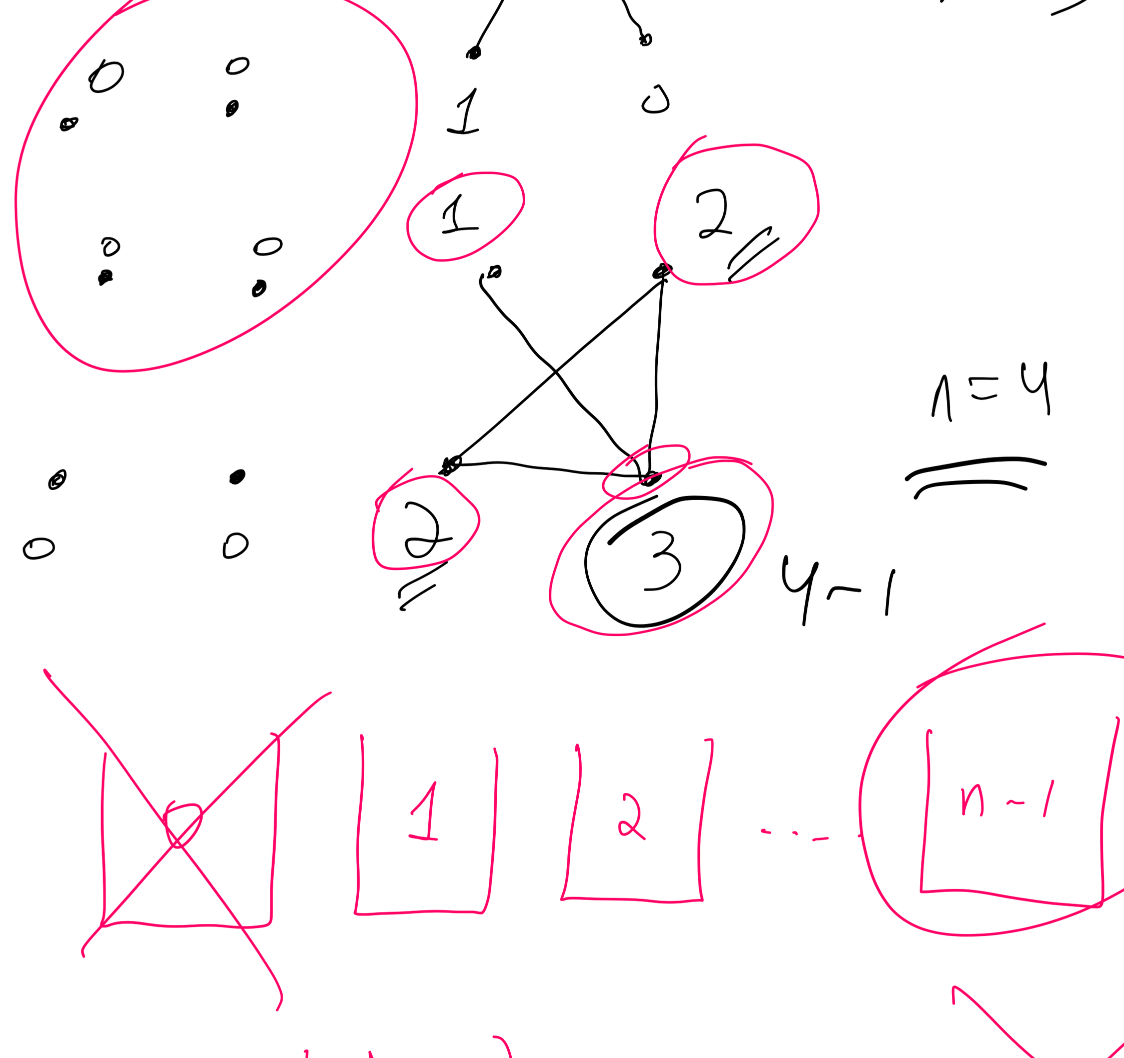
Ex:

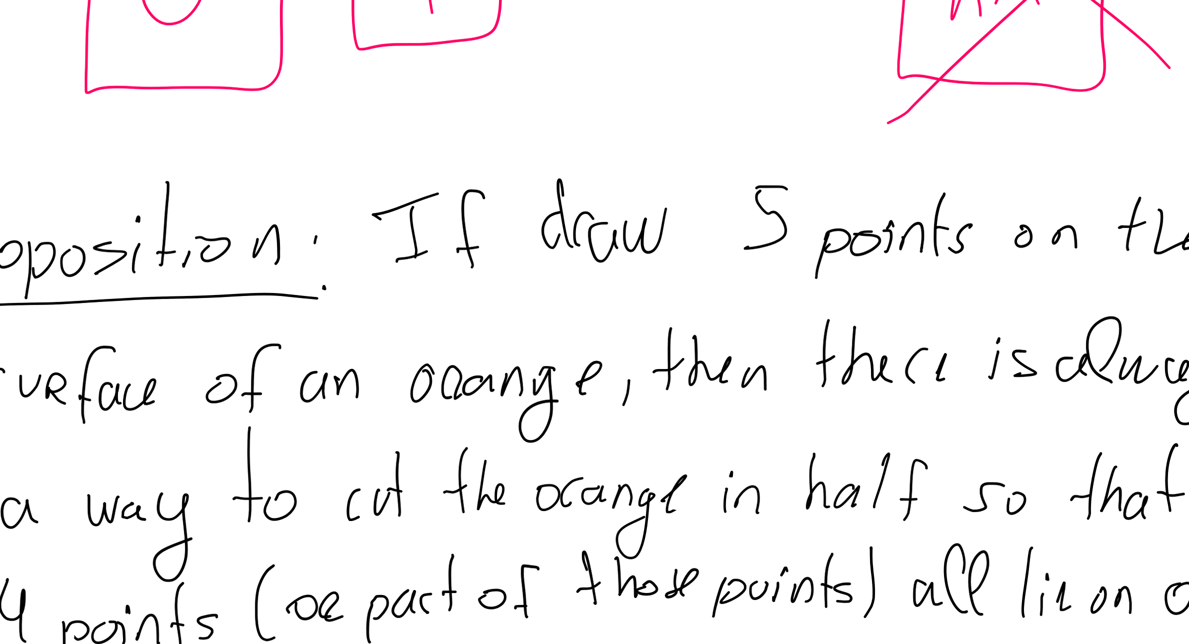


degree of a vertex: # edges touching that vertex

Proposition Given any graph with more than 1 vertex, then the graph contains 2 vertices with same degree.

"n" vertices, $n \geq 2$





Proposition: If draw 5 points on the surface of an orange, then there is always a way to cut the orange in half so that 4 points (or part of those points) all lie on one of the halves.