The contrapositive

$p \to q \iff \sim q \to \sim p$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\sim q \to \sim p$
T	T	T
T	F	F
F	T	T
F	F	T

Ex: Good food is not cheap

If the food is good (p) then it is not cheap (q).

$\sim q \to \sim p$

If the food is cheap then the food is not good

Ex: This class is fun (p), I'll get an A (q). $q \to p \not\Leftrightarrow \sim q \to \sim p$

If I don't get an A then this class is not fun.

Paradox: It's counter intuitive statement.

Russell paradox: $R = \{x ; x \notin x\}$

$R \in R \iff R \notin R$

Zermelo-Fraenkel set theory

5.4 (a) $p \wedge q$ (b) $\sim p$ (c) $\sim p$ (d) $\sim p$ (e) $p \wedge q$ (f) $\sim p$ (g) $p \vee q$ (h) $p \wedge q$

5.5 $\frac{2n^2+5+(-1)^n}{2}$ is prime

n=1; 3 is prime True

n=2

n=3

n=4

n=5; 27 is prime False.

5.6 x is even.

x=2 True x=1 False

5|x

x=10 True x=1 False

5.7 "$\forall$" For all, Every, all, ...

"$\exists$" There is, exists, ...

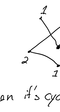
(a) If f is an odd function then f(0) = 0. $f(-x) = -f(x)$, $f(0) = -f(0)$, $2f(0) = 0$, $f(0) = 0$

Every odd function satisfies f(0) = 0.

(b) The equation $x^3 + x = 0$ has a solution.

There is a $x \in \mathbb{R}$ such that $x^3 + x = 0$

$\exists x \in \mathbb{R}; x^3 + x = 0$



5.8 (a) If a group has prime order then it's cyclic.

(b) If two graphs are isomorphic then they have identical degree sequences.

(g) If an integer is not odd then it's even. $\sim(p \to q) \iff p \wedge \sim q$

5.9. (a) A matrix A is invertible if and only if $\det(A) \neq 0$

5.10. (a) $\exists p; \forall q$ primes : $q \leq p$. $\sim(p \wedge q) = \sim p \vee \sim q = \sim p \vee q$, $x^n - n \cdot x^{n-1}$

(b) There is a polynomial that is not differentiable

(c) $xy = 0$ and $x \neq 0$ and $y \neq 0$ $\sim(p \vee q) = \sim p \wedge \sim q$

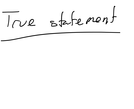
(d) m+n is odd and m isn't odd or n isn't odd.

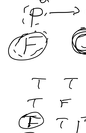
(e) p is prime and $\sqrt{p} \in \mathbb{Q}$

(f) $\exists \varepsilon > 0; \forall N : n > N$ and $|a_n - a| \geq \varepsilon$.

(h) I pass Algebra I and Analysis I this semester and I won't take Algebra II and I won't take Analysis II next semester.

5.12.





If $a = 30°, b = 50°$ then $c = 100°$

converse: If c = 100 then a = 30 and b = 50 (False)

5.16 $p \wedge q \wedge r$

False

$p \to q$

T F

F T or F

T

5.17 "If there is a flying purple in this room then it is riding a centaur."

True statement

5.21 $p \to q$ $q \to p$

If a number is "even" then $n = 2k, k \in \mathbb{Z}$

If $n = 2k$ then n is even

If I was born between April 20th and May 20th then you are taurus.

5.24.

(a) $\forall n \in \mathbb{N} : n^2 \in \mathbb{N}$

We have it follows ...

(b) $\forall r \in \mathbb{R} : \sqrt[3]{r} \in \mathbb{R}$