

Taylor & Karilyn

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0.5

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0.5

Jose & Cameron  
1.5

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Josue & John  
2

4.12

$$3a + 7b$$
$$a = b \Rightarrow 10a, 3a + 7, 7a + 3$$

10, 17, 20, 30, 16, 22, 24, ...

$$n - 3 = 3a + 7b$$
$$n = 3(a + 1) + 7b$$

4.13

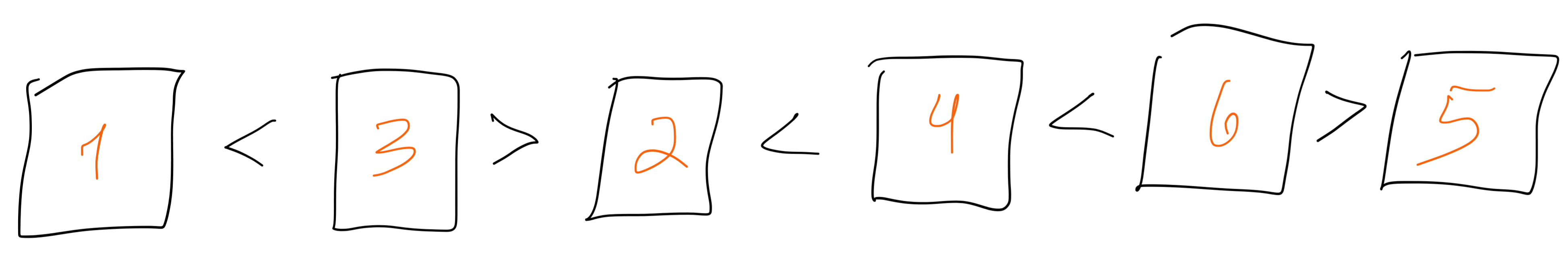
$$2 + 4 + 6 + \dots + 2n = \frac{n \cdot (n + 1)}{2}$$
$$4.2 \Rightarrow \frac{1 + 2 + 3 + \dots + n = \frac{n \cdot (n + 1)}{2}}{\text{or induction}} \Leftrightarrow 2 \cdot (1 + 2 + \dots + n) = n \cdot (n + 1)$$

4.16 (Bernoulli's ineq.)

$$(1 + x)(1 + x)^n \geq 1 + n \cdot x \quad (1 + x)^{n+1} \geq (1 + nx)(1 + x) \geq 1 + (n+1)x$$
$$(1 + x)^{n+1} \geq (1 + nx + nx^2) \geq \frac{1 + x + nx}{1 + x(1 + n)}$$

4.20

1, 2, ..., 6



4.23. (a)  $a_1 = 1, a_2 = 3, n \geq 3$

$$a_n = 2a_{n-1} - a_{n-2}$$

$$a_n = 2n - 1, \quad a_{n+1} = 2a_n - a_{n-1}$$

$$a_{n+1} = 2(n+1) - 1$$

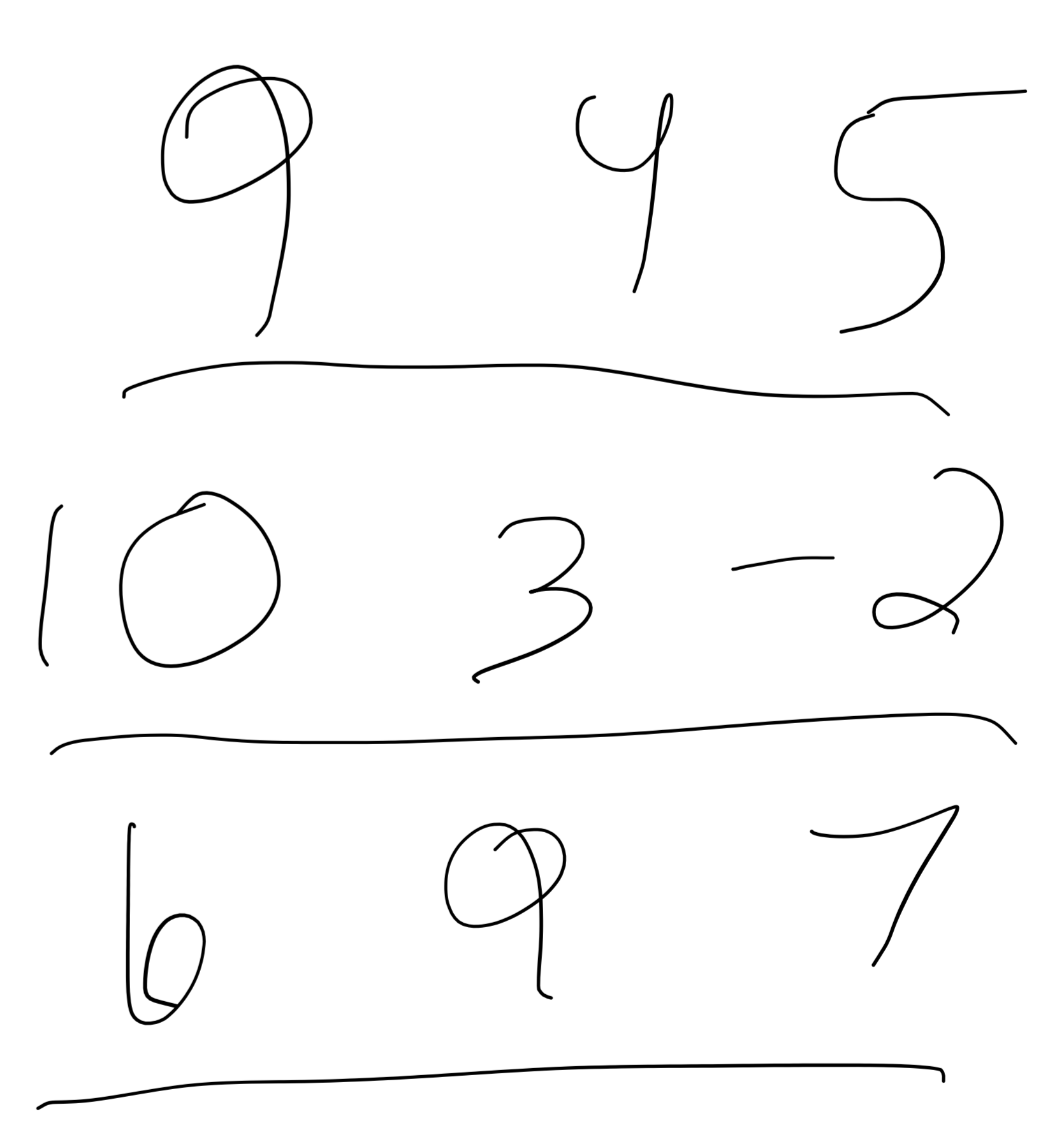
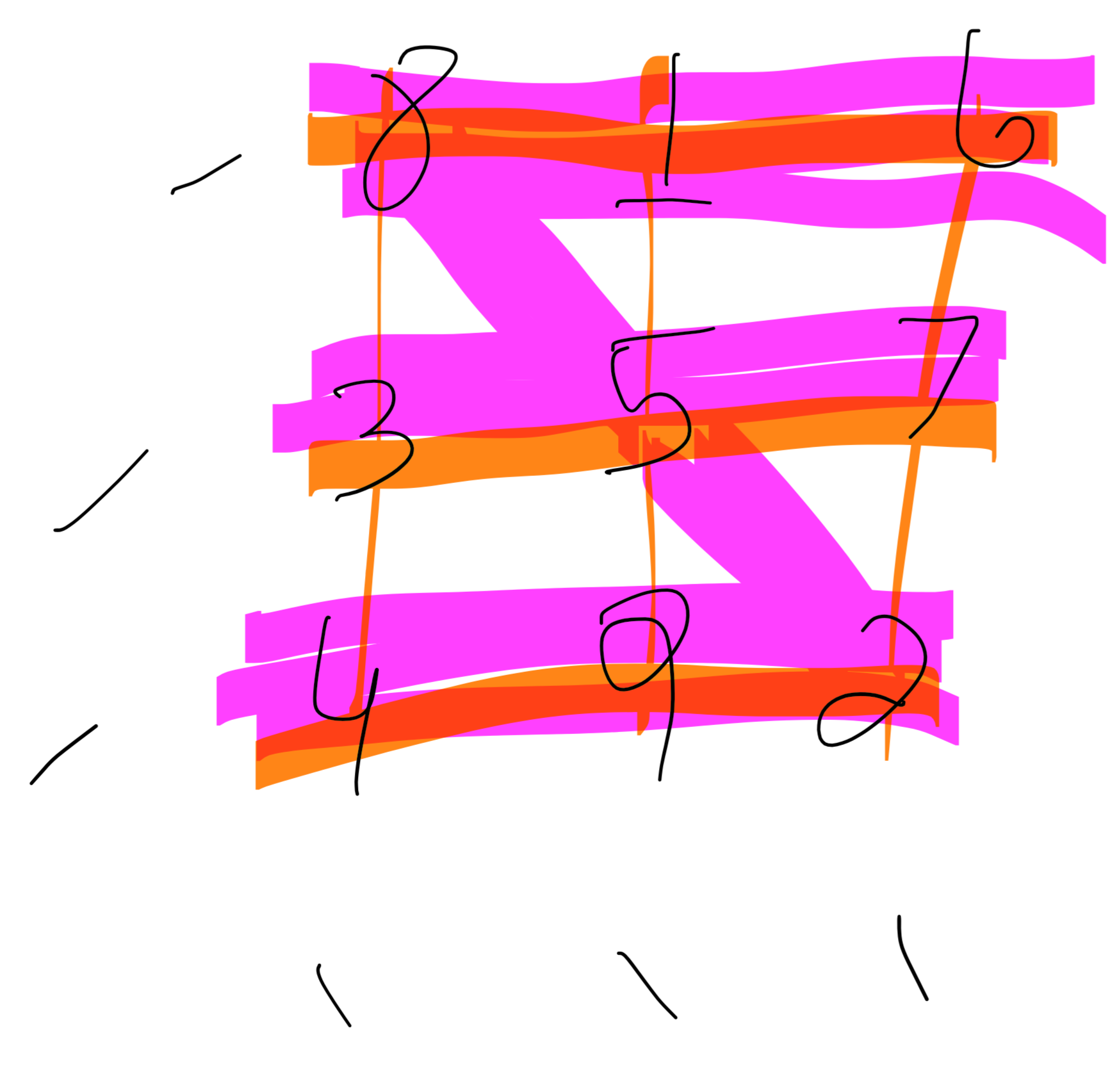
$$a_{n+1} = 2(2n - 1) - [2(n-1) - 1]$$
$$= 4n - 2 - [2n - 3]$$
$$= 2n + 1$$
$$= 2(n+1) - 1$$

4.24  $\forall n \in \mathbb{N}$

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} < 3$$

4.29

3x3



$\forall n \in \mathbb{N} \quad n \geq 2$

there is an  $n \times n$  anti-magic square  
all of whose entries are positive numbers.

