Monday, September 16, 2024 12:38 PM

1.22.

(a) $|x+y| = |x| + |y|$ is false.

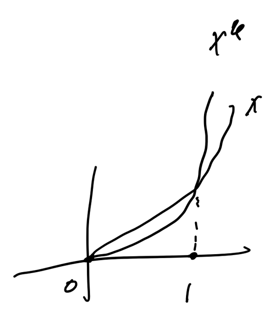
$x = -3,\ y = +5$

$|-3+5|$; $|-3| + |5|$

$2 \neq 8$

$|x+y| \le |x| + |y|$

(b) $x^2 < x^4$



$x = \frac{1}{2} \Rightarrow x^2 = \frac{1}{4}$, $x^4 = \frac{1}{16}$

$\frac{1}{4} < \frac{1}{16}$ is false.

(c) $x, y \in \mathbb{R}$

$|x+y| = |x-y|$ then $y = 0$

$x = 0$; $y = 1$

$|0+1| = |0-1| = 1$

however, $y \neq 0$.

1.17 "Any set of seven integers contains a pair whose sum or difference is divisible by 10"

0	1, 9	2, 8	3, 7	4, 6	5
20	101	22	33, -13	14	15

$n+1$, $n - 2$

$Kn+1$, $n - K+1$

2.38

$a, b \in \mathbb{Z}$

$a|b$ and $b|a \Rightarrow a = b$

$a = 1,\ b = -1$

$1|-1 \quad -1|1 \not\Rightarrow 1 = -1$, $1 \neq -1$

$-1 = 1 \cdot -1 \quad 1 = 1 \cdot -1$

2.39 $n^2 | n \Rightarrow n$ is $-1, 0, 1$.

$n = n^2 \cdot K$

$n = 0$ or $n \neq 0$

$0 = 0$ ✓ $\quad 1 = n \cdot K$

$n = \pm 1$ or $K = \pm 1$

$\gcd(a,b) = d \Leftrightarrow d = pa + qb$

2.34. $\frac{a}{b}$ is in reduced form ; $\gcd(a,b) = 1$

$\frac{21n+4}{14n+3}$ is in reduced form.

$1 = p(21n+4) + q(14n+3)$

$1 = (21p + 14q)n + 4p + 3q$

$21p + 14q = 0 \Rightarrow p = \frac{-14q}{21} = \frac{-2q}{3}$

$4p + 3q = 1$

$4 \cdot \frac{-2q}{3} + 3q = 1$

$\frac{-8q + 9q}{3} = 1$

$q = 3$

$p = -2$

$-42 + 42 = 0$ ✓

$-8 + 9 = 1$ ✓

$1 = (-2)(21+4) + 3(14n+3)$

$\gcd(21+4, 14n+3) = 1$

2.26 $a, b, n \in \mathbb{Z}_+$

$\bar{a} = \{a + k \cdot n ; k \in \mathbb{Z}\}$

$\bar{a} = \bar{b} \Rightarrow \bar{a}^2 = \bar{b}^2$

$\bar{a} = \bar{b}$

$\overline{a-b} = \bar{0}$ $\quad n = 2$

$n | a-b$

$\bar{3} = \{3 + 2k ; k \in \mathbb{Z}\}$

$\overline{a-b} = \bar{0}$

$n | a-b \Rightarrow a - b = k \cdot n$

$\bar{a}^2 = \bar{b}^2 \Leftrightarrow \bar{a}^2 - \bar{b}^2 = \bar{0}$

$n | a^2 - b^2$

$(a-b)(a+b)$

$a^2 - b^2 = (a-b)(a+b) = K \cdot n \cdot (a+b)$

$n | a^2 - b^2$

2.13

$$|x| \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

$|x \cdot y| = |x| \cdot |y|$

$$|x \cdot y| \begin{cases} xy, & xy \ge 0 \\ -xy, & xy < 0 \end{cases} \equiv |x| \cdot |y| \begin{cases} xy, & xy \ge 0 \\ -xy, & xy < 0 \end{cases}$$

xy , $-xy$

xy , $x \ge 0, y \ge 0$

$-xy$, $x < 0, y \ge 0$

$-x \cdot -y$, $x < 0, y < 0$

$-xy$, $x \ge 0, y < 0$

2.15 $n \in \mathbb{Z}_+$

$4 \mid 1 + (-1)^n (2n-1)$

$n = 2K$ $\quad n = 2K+1$

$1 + (-1)^{n=2K} (4K-1)$ $\quad 1 + (-1)^{2K+1} (4K+2-1)$

$1 + 4K - 1$ $\quad 1 - (4K+1)$

$4K$ $\quad -4K$

2.36. Every odd integer is a difference of two squares

$11 = 6^2 - 5^2$

$13 = 7^2 - 6^2$

$15 = 8^2 - 7^2$

$\vdots$

$2K+1 = a^2 - b^2$ $\quad K^2 + 2K + 1$

$= (K+1)^2 - K^2$

$= (K+1-K) \cdot (K+1+K)$

$= 1 \cdot (2K+1)$

2.37 $n \in \mathbb{Z}_+$ $\quad$ Hint : $n! = n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$

⊢ There exists "n" consecutive integers none of which are primes.

$n!+2$, $n!+3$, ... $n!+n$